

Provably High-Quality Solutions for the Liquid Medical Oxygen Allocation Problem

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Abstract

Oxygen is an essential life-saving medicine used in several indications at all levels of healthcare. During the COVID-19 pandemic, the demand for liquid medical oxygen (LMO) has increased significantly due to the occurrence of lung infections in many patients. However, many countries and regions are not prepared for the emergence of this phenomenon, and the limited supply of LMO has resulted in unsatisfied usage needs in many regions. In this paper, we formulated a linear programming model with the objective to minimize the unsatisfied demand given the constraints of supply and transportation capacity. The decision variables are how much LMO should be transferred from a place to another at each time interval using a specific number of vehicles. Multiple storage points are added into the network to allow for more flexible allocation strategies. The proposed model is implemented in India with real-world LMO supply and demand data as a case study. Compared to the manually designed allocation strategy, the proposed model successfully reduces the unsatisfied demand.

1. Introduction

Oxygen supplementation is an integral part of the management of various respiratory diseases. Its importance has been underscored during the COVID-19 pandemic. Both acute and severe manifestations of COVID-19 can be managed with oxygen. The supply of oxygen can take many forms: oxygen cylinders, oxygen concentrators or liquid oxygen [1].

Data from China suggest that while most COVID-19 patients are mildly ill (40%) or moderately ill (40%); about 15% of them are severely ill requiring oxygen therapy and 5% will become very ill. Intensive care unit treatment is required. Additionally, most critically ill patients with COVID-19 require mechanical therapeutic ventilation. For these reasons, COVID-19 treatment facilities should be equipped with pulse oximeters, fully functional oxygen systems, including disposable oxygen delivery ports. [2] For the estimated 500,000 people infected with COVID-19 in low- and middle-income countries (LMICs) during the pandemic wave in mid-2021, 1.1 million cylinders will be required per day. Still, a large number of patients do not have access to oxygen [3]. Medical oxygen shortages are being reported in low- and middle-income countries, mainly from South America, Africa and Asia, where demand for oxygen has increased 100 to 200 times during the pandemic. [4] In Yemen, a low- and middle-income country with a COVID-19-related death rate as high as 19%, the country has also reported a medical oxygen shortage crisis. [5]

Therefore, how to allocate the limited oxygen resources has become an important issue. Some researchers have studied this related logistics problem. The work of F. Costantino et al. [6] designed and analyzed a simulation model for the medical oxygen supply chain. Their focus is on simulating and reproducing the supply chain, rather than optimizing the supply network. In the work of M. Kumar et al. [7], a genetic algorithm is applied to the proposed logistics optimization model to generate routes for two VRP models with time windows for simultaneous pickup and delivery and two pickup strategies for full and partial pickup. Routes are then selected based on a comparison of a set of economic and environmental cost factors.

The research focus of this paper is to find an exact optimal solution for the dynamic network based on the historical data of the real situation. What we use here is a Linear Programming optimization model to solve the problem. In this paper, we develop an exact solution approach for the liquid medical

oxygen (LMO) logistics problem in which we assume perfect information about the LMO demand over the entire service period, i.e. we assume a clairvoyant decision maker. Of course, perfect information on oxygen demand is far from reality, but the obtained solutions can (1) provide valuable insights into high-quality solution characteristics, which can inform online scheduling algorithm design, and (2) Benchmarks are provided to evaluate the quality of online scheduling algorithms.

The remainder of this paper is organized as follows: Section 2 describes the problem setting. The formulation is developed in Section 3 to find optimal solutions. Section 4 conducts result analysis on the performance of this optimization model. Section 5 draws conclusions and provides an outlook for future works.

2. Problem Setting

In this section, we introduce a stylized model of oxygen logistics network, with the goal of formalizing what we consider to be their main structural features: multiple source points and storage points, dynamic oxygen demand of multiple district points, number of trucks and containers, delivery capacity of transport. As a first attempt to study such systems, we postulate a deterministic dynamic framework.

Let S be a set of source points (the locations that can produce and supply LMO), and let each source point $s \in S$ have an associated location l_s . Let D be a set of district points (the locations that consume LMO), and let each district point $d \in D$ have an associated location l_d , and an exact need for LMO NO_d . Let W be a set of storage points, where each storage point $w \in W$ has an associated location l_w . The total service time of this logistics network is six days. The oxygen production of the source point and the oxygen demand of the district point are different every day, that is, they change with time. In this paper, we assume that all information about S , D and W for the whole service time period is known.

For each source point, there are dedicated containers and trucks available for its use. The container is running on the railway, which has higher capacity and longer daily running time, but its total transportation distance is longer. The truck is running on the road, its capacity is small, but the speed is fast, and the total transportation distance is short. Whether trucks or containers, their task is to transport LMO from source points to district points for use or storage points for storage. And they need to return to the original source point after completing the delivery task. At the same time, their quantity is limited. Once there are no extra trucks or containers available, source points will lose the ability to deliver LMO. At the same time, we limit that source points do not have the conditions to store LMO. All LMO produced by source points must be sent out immediately.

For each source point, there are dedicated containers and trucks available for its use. These trucks and containers have the same properties as mentioned above, but their task is to transport the LMO stored in the storage points to the district points for use. Similarly, after these trucks and containers complete the delivery tasks, they have to return to the original storage points before they can start the delivery tasks in sequence. In this paper, there is no upper limit to the amount of LMO stored in storage points. We need to note that each storage point belongs to a specific state $State_w$. This means it will only provide LMOs for district points within the state.

For each district point, they will obtain LMO from source point and storage point in the state. If there is a situation where the LMO needs are not met, then there will be a penalty. The objective function of our optimization model is to minimize the sum of penalties of all district points. That is, the sum of unsatisfied demands in the network is as small as possible.

For clarity, the timeline of a delivery task is shown in Fig. 1. Fig. 2 demonstrates the logistics network of LMO.

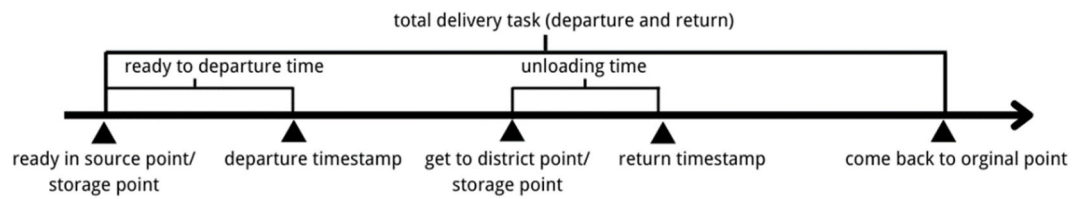


Fig. 1. Timeline of a delivery task

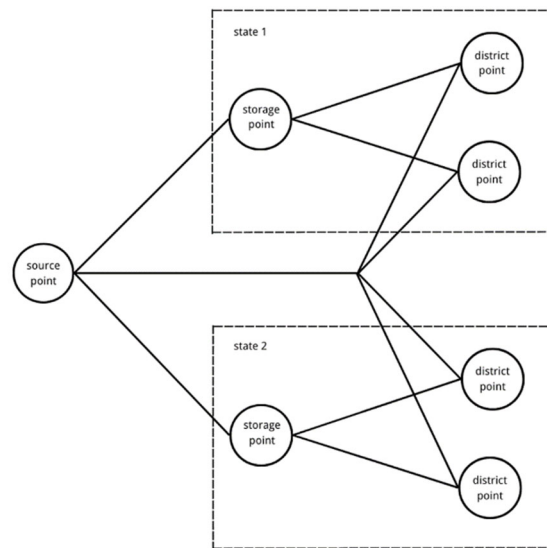


Fig. 2. Logistics network of LMO

3. Mathematical Formulation

In this section, we will present and explain the formulation of this optimized model. Since there are many variables and parameters involved in this part, for clarity, we list the notation and related descriptions in Table I.

TABLE I. NOTATION AND DESCRIPTION

Notation	Description
S	Set of all source points
D	Set of all district points
W	Set of all storage points
s	Source point in set S
d	District point in set D
w	Storage point in set w
NO_d	Demand of LMO in district point d
N_s	Total number of all source point in set S
N_d	Total number of all district point in set D
N_w	Total number of all storage point in set W
$N_{horizon}$	Total number of days in the service time period
X_s	Amount of LMO supplied in source point s
W_w	Amount of LMO stored in storage point d
t	Timestamp in the total service time

Due to the large scale of this problem and the large number of variables, we cannot define them as separate decision variables. Therefore, in this model, we turn them into variable matrix for calculation, which greatly reduces the time of modeling and calculation. O represents the amount of LMO delivered by different pathways. Y represents the number of trucks or containers executing the delivery task at a certain time. Z represents the number of unused trucks or containers that stop at a certain location at a certain time. R represents the number of trucks or containers that return to the origin to execute the next delivery task after completing the delivery task at a certain time. For the calculation of the R variable, we will discuss it in detail below. $district_fulfilled$ represents the amount of LMO consumed by each district point at a certain time. $penalty_district$ represents the penalty that each district point receives due to unsatisfied demands at a certain time. With these decision variables and parameters, we define our formulation as the following:

First of all, we need to limit the amount of LMO produced by the source point to not exceed its daily production limit:

$$X_{i,t} \leq source_list_{i,t} \quad (1)$$

The next thing we have to consider is the upper limit of the LMO amount for source points and storage points transportation. The amount of oxygen they transport each day should be less than the limit of LMO that can be transported by trucks and containers from that point:

$$O_rail_source_{i,j} \leq Y_{rail_source_{i,j}} * cap_cont \quad (2)$$

$$O_road_source_{i,j} \leq Y_{road_source_{i,j}} * cap_truck \quad (3)$$

$$O_rail_storage_{w,j} \leq Y_{rail_storage_{w,j}} * cap_cont \quad (4)$$

$$O_road_storage_{w,j} \leq Y_{road_storage_{w,j}} * cap_truck \quad (5)$$

In this model, at a certain point the input and output of the LMO should be in balance. No LMO disappears or appears out of nowhere. Therefore, we need to add an LMO level constraint to the source point and storage point. It should be noted that for the first day of service, we have to consider the storage balance separately, because the storage point does not store any oxygen before that:

$$X_{i,t} = \sum_j O_rail_source_{i,j} + \sum_j O_road_source_{i,j} \quad (6)$$

$$\begin{aligned} W_{w,t-1} + \sum_i O_rail_source_{i,w} + \sum_i O_road_source_{i,w} \\ = W_{w,t} + \sum_j O_rail_storage_{w,j} + \sum_j O_road_storage_{w,j} \end{aligned} \quad (7)$$

After considering the LMO balance of source points and storage points, we have to start to consider the limitations of district points. First of all, we need to ensure that the LMO consumption of each district point is equal to the amount of LMO input to this district point. At the same time, we also need to limit the LMO consumption of each district point to be less than the demand of the day.

Excessive LMO consumption is meaningless:

$$district_fulfilled_{j,t} \leq district_list_{j,t} \quad (8)$$

$$\begin{aligned} district_fulfilled_{j,t} &= \sum_i O_{rail_source_{i,j}} + \sum_i O_{road_source_{i,j}} + \sum_w O_{rail_storage_{w,j}} \\ &+ \sum_w O_{road_storage_{w,j}} \end{aligned} \quad (9)$$

Through the above work, we have completed the modeling of an LMO logistics network that does not consider the delivery capacity, ensuring the balance of LMO in each location. But this is obviously not enough. We need to consider the limitations of trucks and containers in actual transportation. We have mentioned in the section of problem setting that for all vehicles, they need to return to the origin after completing the delivery work. In traditional distribution network modeling, departure and return trip are often modeled separately. But in this case, we need to add many dimensions to these variables, including the belonging location to which these vehicles belong. Here, a new formulation method is proposed. We set a binary parameter matrix which is precalculated to indicate whether a vehicle with diversity work is able to return at a time interval or not. This parameter matrix is the return matrix N . Here we set up $N1$ matrix for container from source points to district points as an example:

$$\begin{aligned} \text{if } t_1 + travel_time_rail_{i,j} = t_2: \\ N1_{t_1 * N_{source+i,j}, t_2} = 1 \end{aligned} \quad (10)$$

In addition to the $N1$ matrix, we have five other similar parameter matrices to indicate whether the vehicle can return to the origin and start the next delivery task in various situations. $N2$ matrix is for trucks from source points to district points. $N3$ matrix is for containers from source points to storage points. $N4$ matrix is for trucks from source points to storage points. $N5$ matrix is for containers from storage points to district points. $N6$ matrix is for trucks from storage points to district points.

With these pre-calculated binary parameter matrices, we can calculate how many vehicles will return to the origin at a certain time. We use decision variable matrix R to represent these quantities. Here we set up R_{rail_source} to signify the number of containers that will come back to source points at certain time as an example:

$$\begin{aligned} R_{rail_source_{i,t_2}} &= \sum Y_{rail_source_{t_1 * N_{source+i,j}}} * N1_{t_1 * N_{source+i,j}, t_2} \\ &+ \sum Y_{rail_source_{t_1 * N_{source+i,w}}} * N3_{t_1 * N_{source+i,j}, t_2} \end{aligned} \quad (11)$$

In addition to the R_{rail_source} matrix, we have three other similar parameter matrices to signify the number of vehicles can return to the origin and start the next delivery task in various situations. R_{road_source} signify the number of trucks that will come back to source points at certain time. $R_{rail_storage}$ signify the number of containers that will come back to storage points at certain time. $R_{road_storage}$ signify the number of trucks that will come back to source points at certain time.

After that, we need to consider the balance of traffic flow. For each location, the input and output

of the vehicle should be consistent. At the same time, we have to single out the first day for consideration. Because on the first day, no vehicles that were at the point before the service time were available:

$$S_{cont_source_{i,t}} + R_{rail_source_{i,t}} + Z_{rail_source_{i,t-1}} = \sum_j Y_{rail_source_{i,j}} + Z_{rail_source_{i,t}} \quad (12)$$

$$S_{truck_source_{i,t}} + R_{road_source_{i,t}} + Z_{road_source_{i,t-1}} = \sum_j Y_{road_source_{i,j}} + Z_{road_source_{i,t}} \quad (13)$$

$$S_{cont_storage_{w,t}} + R_{rail_storage_{w,t}} + Z_{rail_storage_{w,t-1}} = \sum_j Y_{rail_storage_{w,j}} + Z_{rail_storage_{w,t}} \quad (14)$$

$$S_{truck_storage_{w,t}} + R_{road_storage_{w,t}} + Z_{road_storage_{w,t-1}} = \sum_j Y_{road_storage_{w,j}} + Z_{road_storage_{w,t}} \quad (15)$$

Till now, we have basically completed the modeling of the river network. But it should be noted that we said that storage points will only serve district points in the state. Therefore, we need to add constraints to it to ensure the realization of this premise.

Finally, we still have constraints on the calculation of the penalty function. With this constraint, we can set the objective function of this optimization model, which is to minimize the sum of the penalties of each district point:

$$Objective\ Function = \min \left(\sum penalty_district \right)$$

4. Computational Study

In this computational study section, two indexes are adopted to assess the algorithms and the dynamic system's performance: 1) The sum of penalty of all district points. 2) The solving time of the optimization model.

In order to prove the effectiveness of this model, we use historical real data from India for research. In an instance, 79 source points, 49 storage points, and 109 district points. In Fig. 3, we can see that in most cases, the LMO consumption of district points remains basically stable. Fig. 4 presents the total supply of LMO per day. This speaks to the ample supply of LMO. Fig. 5 presents the total consumption of LMO per day. We can see that as the service time progresses, the oxygen consumption in the network continues to increase and tends to be stable. This illustrates the high efficiency of our optimized model when serving for a long time.

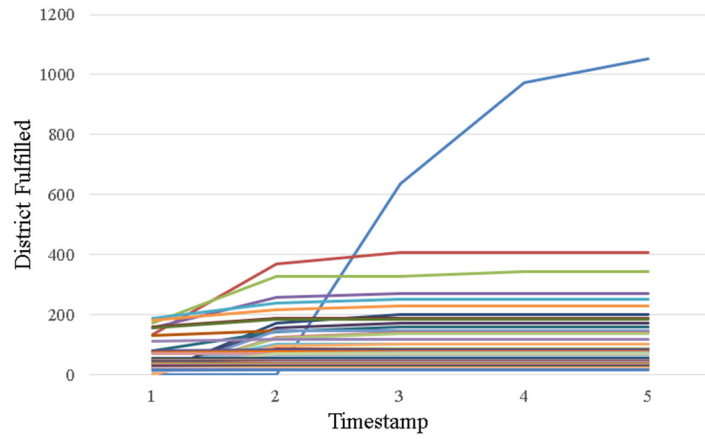


Fig. 3. District point LMO consumption curve

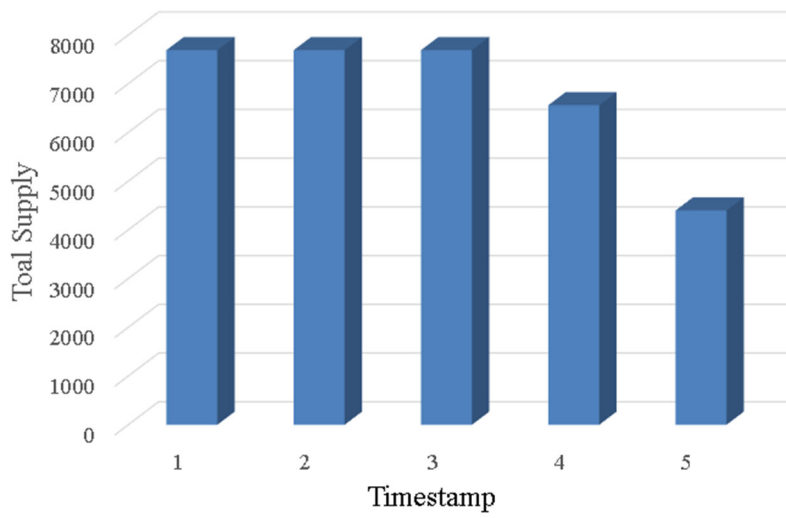


Fig. 4. Column chart of total LMO supply

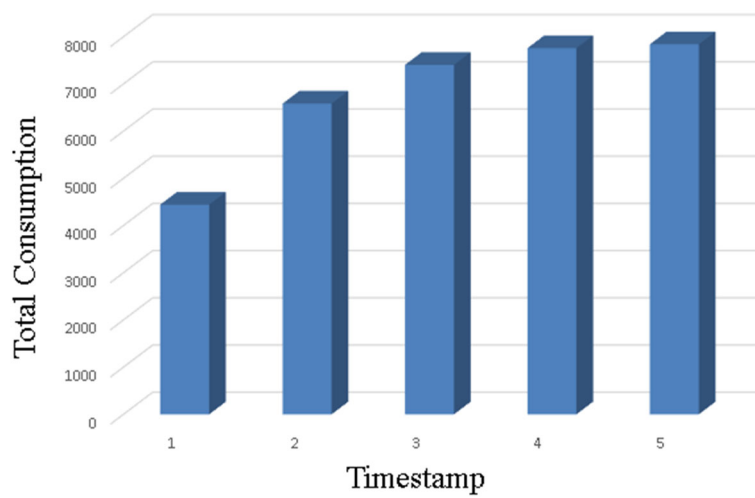


Fig. 5. Column chart of total LMO consumption

At the same time, we also compared with the manually designed allocation strategy, we can find that the proposed model reduces the sum of penalty of all district points by 20%. With the help of pre-

calculated return matrix, the departure and return trips of a vehicle can be modeled as whole (instead of separately in the original formulation), which greatly reduces the problem scale by eliminating the number of decision variables by 90%. The solving time decreased from 3 minutes to 25 seconds.

5. Conclusion

This paper proposes a provably high-quality solutions for the liquid medical oxygen allocation problem. We can get an accurate optimal result of the oxygen allocation and how much oxygen should be produced in the plants every day. Concurrently, a new formulation method for this problem is proposed and proved to be able to effectively reduce the time of modeling and solving.

This optimization model has some limitations. First of all, in reality, it is difficult for us to have a perfect grasp of information. We can only obtain limited information for optimization at a certain decision point. At the same time, this model obtains a linear programming result. Although this method is reasonable (discrete optimization is difficult and the amount of calculation is too large), we still need more work to prove it.

In the future, we will try to use the rolling-horizon method to achieve real-time optimization. At the same time, it is also necessary to perform discrete optimization on small-scale instances and compare the results with that of our current linear programming and see whether it is possible to find discrete optimal solutions from our model.

6. Reference

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